LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **NOVEMBER 2023**

PST 2501 – ESTIMATION THEORY

Date: 31-10-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION -A

Answer ALL the questions . **10 x 2 = 20 Marks**

1.Define statistic and provide an example.

2. Let $X_1,X_2,,,X_n$ be i.i.d. $B(1,\theta)$, $0 < \theta < 1$. Obtain a sufficient statistic for θ.

3.Let X_1,X_2 be i.i.d. random variables from Poisson $P(\theta)$, $\theta > 0$. Show that $T = X_1 + 2X_2$ is not sufficient for θ.

4. Define exponential family of distributions.

5.Define loss function and give an example.

6.When a family of distributions is called complete?

7. When a statistic is called ancillary? Give an example.

8.Write any two properties of M.L.E.

9. Define CAN estimator and provide an example.

10.Write a note on Jackknife method.

SECTION -B

Answer any FIVE questions. **5 x 8 = 40 Marks**

11. If $\{\delta n\}$ is a sequence of UMVUEs and δn converges to δ almost surely as $n\to\infty$, then show that δ is UMVUE.

12.State and prove the Neyman-Factorization theorem.

13. Let $X\sim U(0,\theta)$, $\theta > 0$. Assume that the prior distribution of Θ is $h(\theta) = \theta$ e $-\theta$, , $\theta> 0$. Find Bayesian estimator of θ if loss function is (a) squared error and (b) absolute error. (4+4)

14. (a) Establish the invariance property of CAN estimator. (4)
(b) State and prove Basu's theorem. (4)

15. Show that $\{N(\theta,1) , \theta \in R\}$is complete.

16.Let $X_1,X_2,\ldots X_n$ be a random sample of size n from $P(\theta)$, $\theta > 0$.
(a) Obtain MVBE of θ. (6)
(b) Suggest MVBE of $a\theta + b$, where a and b are constants and $a\neq 0$. (2)

17. (a) State and prove Lehmann-Scheffe theorem. (4)
(b)Let $X_1,X_2,\ldots X_n$ be a random sample of size n from
$f(x;\theta) = \exp\{ -(x-\theta)\}$, $x \geq \theta$, zero elsewhere. Find UMVUE of θ. (4)

18. (a) Show with an example that MLE is not sufficient. (4)
(b) Let $X_1,X_2,...X_n$ be a random sample from $N(\theta,1)$, $\theta \in R$. Find a consistent estimator for θ. (4)

SECTION-C

Answer any TWO questions **2 x 20 = 40 Marks**

19.(a) Let X be a discrete random variable with the probability mass function

$P_\theta(x) = \theta$, x = -1 and $P_\theta(x) = (1-\theta)^2\,\theta^x$, x= 0,1,2… , $0 < \theta < 1$.

Find (i) U_0 (ii) U . (5 +5)

(b) Let X ~ DU{1,2,3,…N} , N = 2,3,4… Find UMRUE using calculus approach. (10)

20. (a)State and prove the necessary and sufficient condition for an estimator to be UMVUE using uncorrelated approach. (14)

(b) Show that UMVUE, if exists, is unique. (6)

21. Let $X_1, X_2, \ldots X_n$ be a random sample of size n from $N(\mu, \sigma^2)$,where μ *and* σ^2 are unknown . Find UMVUE of (i) μ (ii) σ^2 and (iii) μ^2 / σ^2 . (5+5+10)

22.(a) Let $X_1, X_2, \ldots X_n$ be a random sample from $N(\theta,1)$, $\theta \in R$. Show that sample mean and variance are independent using Basu's theorem. (12)

(b) Establish the invariance property of MLE and illustrate with an example. (8)

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